

The limited zone of motion and optimal trajectory of industrial robot

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ABSTRACT: This paper presents the method for determining the optimal trajectory based on limited condition of dynamics to define the limited motion zone and the characters of all of trajectories in that zone. Its application to determination for the optimal trajectories of time and energy for periods in motion cycle of manipulator with two rotary links.

KEYWORDS: Robot, Limit, Motion, Trajectory, Optimal, Time, Energy.

I. INTRODUCTION

The motion control of an industrial robot from a given initial state to a given end state can be performed in an infinite number of different motion paths, so it is necessary to choose the optimal motion trajectory according to the goal, is set. The application of classical optimal control theories to this problem still faces many difficulties because these theories are only necessary but not sufficient conditions, the obtained results are relative, may not be optimal. In recent application, it is common to use computational software programs to determine the target braking value for a limited number of possible scenarios and then select a value from which in a similar sense.

To overcome the above limitations, this paper will propose a method to determine the optimal state of motion trajectory based on the dynamic limit to determine the motion limit domain and the nature of all orbitals. The possible states of motion are exempt from that limitation.

II. SPECIAL TRANSFORMATION LIMITED DOMAIN

The robot motion control problem is set as follows. For a robot object, the dynamics equation in general form is:

$$A(q)\ddot{q} + B(q, \dot{q})\dot{q} + C(q) = u \quad (1)$$

With the limited condition:

$$u_{\min} \leq u \leq u_{\max}$$

Determine the effect u such that the state of the object changes from the state point initially $M(q_M; \dot{q}_M)$ has coordinates q_M , and the speed is \dot{q}_M to the final state point $N(q_N; \dot{q}_N)$ with coordinates q_N , and speed \dot{q}_N , to reach a predefined target.

The change of the state of motion at a point is represented by the relationship of the change in motion $d\dot{q}$ and dq , we have:

$$\frac{d\dot{q}}{dq} = \frac{\frac{d\dot{q}}{dt}}{\frac{dq}{dt}} = \frac{\ddot{q}}{\dot{q}} \text{ or } \frac{d\dot{q}}{dq} = \frac{\ddot{q}}{\dot{q}} \quad (2)$$

The geometrical meaning of (2) is the slope at a point of the motion state trajectory. At each given state point (q, \dot{q}) , from (1) we have $A(q), B(q, \dot{q})\dot{q}$ and $C(q)$ definite. Since u has a limit, $u_{\min} \leq u \leq u_{\max}$, therefore q also has a limit, $\dot{q}_{\min} \leq \dot{q} \leq \dot{q}_{\max}$ and the slope at a point of the moving state orbital also have a limit:

$$\frac{\ddot{q}_{\min}}{\dot{q}} \leq \frac{d\dot{q}}{dq} \leq \frac{\ddot{q}_{\max}}{\dot{q}} \quad (3)$$

Therefore, the motion state limit domain between two points from M to N is limited by the motion state orbitals with \ddot{q}_{\max} and \ddot{q}_{\min} passing through those two points as shown in Figure 1.

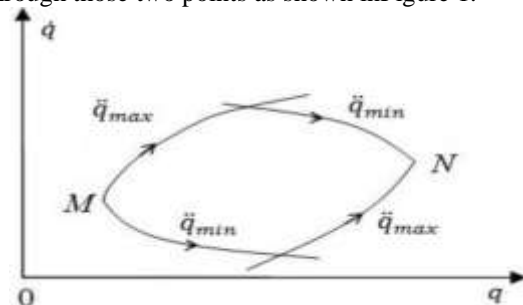


Figure 1. Limiting domain of state orbitals from M to N

III. CHARACTERISTICS OF MOTION STATE ORBITALS AND OPTIMAL MOTION STATE ORBITS

1. State Orbitals.

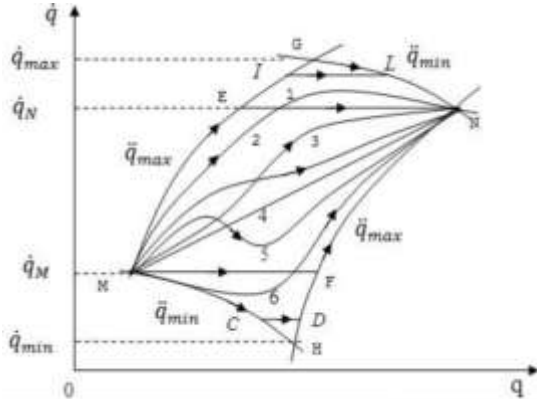


Figure 2. Limiting domain and moving discharge trajectories from M to N

There are infinitely many transition state orbitals from M to N in the domain limited by the orbitals motion states are available at \ddot{q}_{max} and \ddot{q}_{min} passing through those two points, figure 2. The characteristics of the motion state trajectories are different. Below we will consider two important properties, which are the time and energy dissipation of the moving state orbitals.

2. Process time and optimal motion state trajectory time.

The time of the transition from the initial state point (q_0, \dot{q}_0) to the end state point is determined from the expression:

$$T = \int_{q_0}^q \frac{dq}{\dot{q}} = \int_{\dot{q}_0}^{\dot{q}} \frac{d\dot{q}}{\ddot{q}} \quad (4)$$

From (4) we see that the state orbits of motion have a speed of q or an acceleration \ddot{q} greater than time consumption is smaller. From Figure 2, it can be seen that the higher the trajectory of the state of motion, the shorter the time. The shortest-timed motion-discharge trajectory is MGN and is the upper limit of the transition limit region dynamic, has an acceleration of \ddot{q}_{max} and \ddot{q}_{min} then at the screen with the largest time is MHG and is the lower limit of the motion limited region, with an acceleration of \ddot{q}_{min} then \ddot{q}_{max} . Trajectories inside the restricted region of motion (such as 1, 2, 3, 4, 5) have time dissipation in the range of maximum and minimum limit values best.

3. Process energy and energy-optimal motion state trajectories

The expression of energy consumed by the movement from the initial state point (q_0, \dot{q}_0) to the final state point (q, \dot{q}) is given by determined as follows. Substituting in \ddot{q} (2) into (1) and integrating, we have:

$$\int_{q_0}^q A(q)\dot{q}d\dot{q} = \int_{q_0}^q (u - B(q, \dot{q}) - C(q))dq \quad (5)$$

The left statement of (5) is the change of kinetic energy between two points in the state, the right statement is the work of external forces acting (this is the content of the theorem about kinetic energy change). Based on (5), we can calculate the energy dissipated between two state points of the motion process.

From the red line of Figure 3 we have important conclusions. The following:

- All orbitals change monotonically (orbitals 1, 3, 4), have equal energydissipation and equal to the kinetic energy change between M and N.
- Horizontal orbital segments (segments IL, EN, MF, CD) has zero energy dissipation because the kinetic energy transformation is zero. From this, we can conclude that the minimum energy loss is equal to the kinetic energy change (of a simple process) between two state points and that there can be infinitely many orbits with the same energy dissipation. Therefore, in order to obtain the smallest and only energy dissipation trajectory, additional constraints must be met, for example, about the dissipation time.

However, depending on what time interval T is at, the number of energy-optimal orbits is either unique or infinite.

IV. APPLY

We will consider a specific example below:

Let a robot with 2 apertures rotate in the horizontal plane, as shown in Figure 3, with the following parameters. Length of stitches, mm, $l_1 = 600, l_2 = 400$. Mass, kg, $m_1 = 10, m_2 = 6$. Distance center of mass, mm, $l_{c1} = 400, l_{c2} = 300$. Moment of inertia about the correct axis of inertia (passing through the center of mass c), $kg.m^2, J_{c1} = 0.3, J_{c2} = 0.13$. Limited torque, Nm, $\tau_1 = \pm 0.4, \tau_2 = \pm 0.2$. Movement limit $\theta_1 = \pm 152^\circ, \theta_2 = \pm 152^\circ$

Determine the motion state trajectory so that the final impact point of the robot moves in aclosed cycle consisting of 3 stages as follows: From point M with coordinates $x = 1000, y = 0$ and

speed $\dot{x}=0, \dot{y}=0$ to point N with coordinates $x=775, y=-36$ and speed $x=-0.1550m/s, y=0.0452m/s$ with minimum dissipative time; The numerator N moves with constant speed along a straight line to the point H with coordinates $x=447, y=45$; The atom H returns to the point M with the initial state in the time $T=10s$ so that the energy consumption is minimal.

From the given conditions, we can establish the system of kinetic equations of the robot as:

$$\tau_1 = \left[m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c1}^2 + 2l_1 l_{c2} \cos \theta_2) + (J_{c1} + J_{c2}) \right] \ddot{\theta}_1 + \left[m_2 (l_{c2}^2 + l_1 l_{c2} \cos \theta_2) + J_{c2} \right] \ddot{\theta}_2 - m_2 l_1 l_{c2} \sin \theta_2 (\dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2) \quad (6)$$

$$\tau_2 = \left[m_2 (l_{c2}^2 + l_1 l_{c2} \cos \theta_2) + J_{c2} \right] \ddot{\theta}_1 + \left[m_2 l_{c2}^2 + J_{c2} \right] \ddot{\theta}_2 + m_2 l_1 l_{c2} \sin \theta_2 \dot{\theta}_1^2 \quad (7)$$

In which, τ_1, τ_2 is the moment of force acting at the joints. Solving the above system of equations, we get the angular acceleration of the apertures as a function of the control action τ_1 and τ_2 and determine the maximum, minimum and different specific values of $\ddot{\theta}_1$ and $\ddot{\theta}_2$ at each point status. Therefore, we can successively determine the neighboring state points at the time before and then to get the state trajectory of the motion.

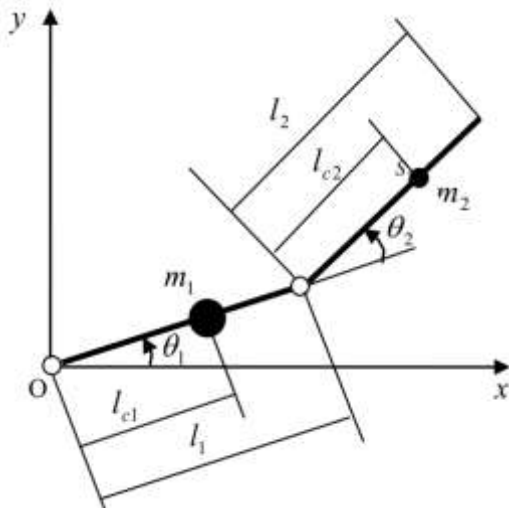


Figure 3. Robot model with 2 guns

The results of the motion trajectories are shown in Figure 4.

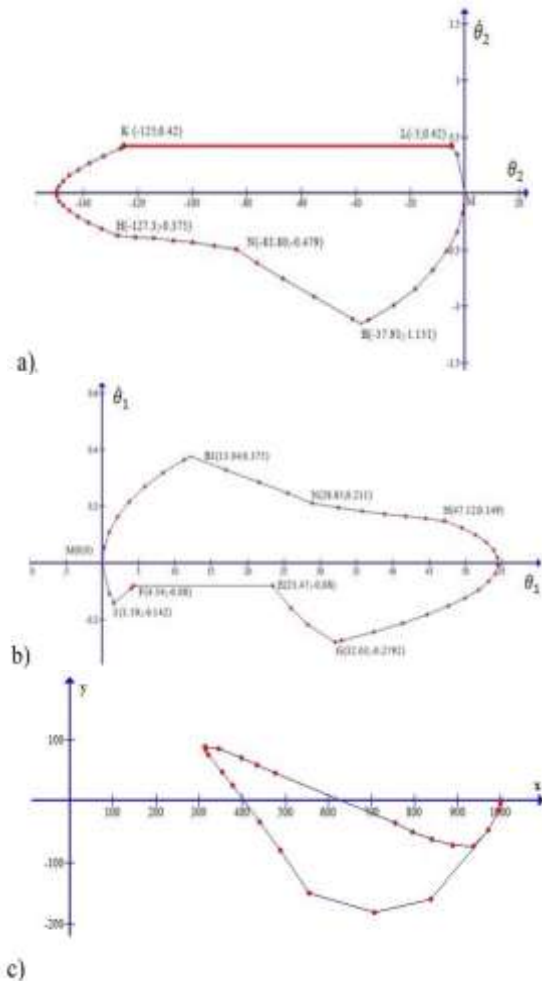


Figure 4. Exhaust trajectory of the apertures: a), b) and the geometrical trajectory of the endpoint: c)

V. CONCLUSION

The paper presents a method to determine the optimal moving state trajectory based on the exemption from the motion state limit and the properties of the orbitals in that limited domain. The red numerator, asserts that the optimal value of the orbital is the largest or the smallest of all possible trajectories under the limited conditions of the problem.

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